Compacton solutions for a class of two parameter generalized odd-order Korteweg–de Vries equations

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We study the fifth-order fully nonlinear $K(m,n,p)$ equations and obtain a class of exact compacton solutions. We find that addition of the fifth-order dispersion term increases the range of the nonlinear parameters m, n, and p for which these compacton solutions are allowed. We consider the Hamiltonian structure and conservation laws of this class of equations. We also study the class of two parameter generalized odd-order equations and obtain the exact compacton solutions and the range of the nonlinearity and dispersion parameters as well as the relation between them. [S1063-651X(98)11003-6]

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I. INTRODUCTION

Recently, Rosenau and Hyman [1] introduced a class of solitary waves with compact support that are termed compactons. Such solitary wave solutions, which vanish outside a finite core region, are solutions of a two parameter family of fully nonlinear dispersive equations $K(m,n)$:

$$u_t+(u^m)_x+(u^n)_{3x}=0, \quad m,n>1. \tag{1}$$

These $K(m,n)$ equations have been used to understand the role of nonlinear dispersions in the pattern formation in liquid drops. These equations have the property that for certain values of the nonlinearity parameters m and n, their solitary wave solutions are compactons, which, like solitons [2], have the remarkable property that after colliding with other compactons, they reemerge with the same coherent shape [1]. The compacton's amplitude depends on its velocity, but, unlike the soliton's, which narrows as the amplitude (velocity) increases, its width is independent of the amplitude and velocity [1,3]. It has been shown that the compacton solutions of Eq. (1) are allowed only for nonlinearity parameters in the range $2\leqslant m=n\leqslant 3$. The upper limit on the parameters m and n is necessary for the compacton solutions to be a solution in the classical sense, i.e., the finite derivatives of the solution at the edges. This allowed range of nonlinearity parameters is determined by the third order dispersive term in Eq. (1). In this paper we consider a fifth-order equation, which we term as the $K(m,n,p)$ equation, of the form

$$u_t+\beta_1(u^m)_x+\beta_2(u^n)_{3x}+\beta_3(u^p)_{5x}=0, \quad m,n,p>1. \tag{2}$$

One of the motivations for studying this higher-order equation is to examine the role of the fifth-order term in the existence of the compacton solutions. Secondly, there has been an increased interest recently on the role of the fifth-order dispersion term on the soliton stability for the usual Korteweg–de Vries (KdV) equation. For example, it has been shown that [4] for the fifth-order equation of the type

$$u_t+\alpha u^p u_x+\beta u_{3x}+\gamma u_{5x}=0 \tag{3}$$

the solitary wave solutions are unstable with respect to the collapse type instabilities if $p\geqslant 4$ for $\gamma=0$ and for $\gamma\neq 0$, i.e., the addition of the fifth-order term, stabilizes the solitons for $p>4$ [4,5]. The exact upper limit of the nonlinearity parameter p in this case is still an open question. In this paper we show that the $K(m,n)$ equation when modified by the addition of the fifth-order dispersion term increases the range of the nonlinearity parameters as compared to that when such a term is absent. In particular, from the exact solutions of Eq. (2) we find that for $\beta_3\neq 0$, the range of the nonlinear parameters $m,n,$ and p for which compacton solutions are allowed is $2\leqslant m=n=p\leqslant 5$, whereas for $\beta_3=0$ [Eq. (1)] the corresponding range is $2\leqslant m=n\leqslant 3$ [1]. Eventually, the number of compacton solutions for integer values of nonlinear parameters increases when higher-order dispersion terms are added. For example, for $\beta_3=0$, there are two compacton solutions for integer values of the nonlinear parameters $m,n=2,3$, whereas, for $\beta_3\neq 0$, there are three compacton solutions for $m,n,p=2,3,5$. Similarly, for the equation with the seventh-order dispersion term, four compacton solutions will be allowed for integer value of the nonlinearity parameters $m,n,p,q=2,3,4,7$. We would like to point out here that the compacton solutions are also allowed for noninteger values of the nonlinearity parameters within their allowed range. We have been able to show that these results are also valid for a class of compacton solutions of the generalized equation with an arbitrary odd-order dispersion term. To show this, we obtain in Sec. IV a class of exact compacton solutions for the generalized two parameter equation with arbitrary odd-order dispersion term. The plan of the paper is as follows: in Sec. II we discuss some exact compacton solutions of Eq. (2). The Hamiltonian structure and the conservation laws of these equations are discussed in Sec. III where we also obtain the mass M and the energy E of the compacton solutions. In Sec. IV, we study the problem for a class of two parameter arbitrary odd-order equations and obtain a class of exact compacton solutions for these generalized equations. From this we obtain a relation between the parameters as

$$\delta=\frac{2n}{k-1}, \tag{4}$$

where k is the nonlinearity parameter, n is the arbitrary odd-order dispersion parameter and δ defines different allowed compacton solutions [see Eq. (7) below]. We conclude in Sec. V.

II. EXACT COMPACTON SOLUTIONS

We seek a solution of Eq. (2) of the form of a traveling wave

$$u(x,t)=u(\xi)=u(x-Dt). \tag{5}$$

In terms of Eq. (5) we can write Eq. (2) after one integration as

$$-Du+\beta_1(u^m)+\beta_2(u^n)_{2\xi}+\beta_3(u^p)_{4\xi}=0\,. \tag{6}$$

Using the ansatz

$$u(\xi)=A\ \cos^{\delta}(B\xi) \tag{7}$$

for $|B\xi|\leqslant\pi/2$ and $u(\xi)=0$ otherwise, in Eq. (6), we obtain a class of one continuous parameter family of compacton solutions, where

$$\delta=\frac{4}{p-1}, \tag{8}$$

$$A^{(p-1)}=\frac{D}{\beta_3B^4p\,\delta(p\,\delta-1)(p\,\delta-2)(p\,\delta-3)}, \tag{9}$$

$$B^2=\frac{2(p^2\delta^2-2p\,\delta+2)}{[p\,\delta(p\,\delta-2)]^2}\,\frac{\beta_1}{\beta_2}, \tag{10}$$

$$\beta_3=\frac{1}{\beta_1}\left[\frac{\beta_2p\,\delta(p\,\delta-2)}{2(p^2\delta^2-2p\,\delta+2)}\right]^2, \tag{11}$$

for $2\leqslant m=n=p\leqslant 5$. Note that the compacton solutions [Eq. (7)] are also allowed for the continuous values of the nonlinearity parameter within the range given above. Some of the typical compacton solutions for the integer values of the nonlinearity parameters are given below:

$$K(2,2,2)\ \text{equation:}\quad u(\xi)=A_2\ \cos^4(B_2\xi),\quad \text{for}\quad |B_2\xi|\leqslant\pi/2, \tag{12}$$

$$K(3,3,3)\ \text{equation:}\quad u(\xi)=A_3\cos^2(B_3\xi),\quad \text{for}\quad |B_3\xi|\leqslant\pi/2, \tag{13}$$

$$K(5,5,5)\ \text{equation:}\quad u(\xi)=A_5\cos(B_5\xi),\quad \text{for}\quad |B_5\xi|\leqslant\pi/2, \tag{14}$$

where A_p and B_p for $p=2$, 3, and 5, respectively, can be obtained from Eqs. (8)–(11). Thus we find that within the allowed parameter range $2\leqslant m=n=p\leqslant 5$ there are three compacton solutions for the integer values of the nonlinearity parameters for the fifth-order equation where as there are only two compacton solutions for the parameters m,n in the range $2\leqslant m=n\leqslant 3$ for the third-order equation [Eq. (1)] [1]. Thus the addition of the fifth-order term not only increases the nonlinearity parameter range but also increases the number of allowed compacton solutions. In Sec. IV we show that this is true for the addition of any arbitrary odd-order dispersion term.

III. CONSERVATION LAWS

A conservation law associated with equations of the form as Eq. (2) can be written as

$$\frac{\partial Q}{\partial t}+\frac{\partial X}{\partial x}=0. \tag{15}$$

This means that Q is conserved in Eq. (2) if we can transform it in the form as Eq. (15). From Eq. (2) we can immediately find one conservation law for which

$$Q=u,\quad X=\beta_1u^m+\beta_2(u^n)_{2x}+\beta_3(u^p)_{4x}\,. \tag{16}$$

We could not find any other conservation laws of this fifth-order equation. This is in contrast to the third-order equation [Eq. (1)] for which there are two conservation laws [1]. To see why there are no other conservation laws to Eq. (2), we note that Eq. (2) is not derivable from a Lagrangian and hence does not possess the usual conservation laws of mass, energy, etc., that are associated with KdV type equations $(n,p=1)$. Hence, for the equation of the same type as Eq. (2), where nonlinearity parameters n,p are $\geqslant 2$, we consider a different fifth-order equation, which can be derived from the Lagrangian, leading to other conservation laws. For this we consider the Lagrangian

$$L=\int\mathcal{L}dx=\int dx\left[\frac{1}{2}\phi_x\phi_t-\delta\frac{(\phi_x)^{a+1}}{a+1}-\alpha(\phi_x)^b(\phi_{2x})^2-\beta(\phi_x)^c(\phi_{2x})^4-\gamma(\phi_x)^d(\phi_{3x})^2\right], \tag{17}$$

which leads to the generalized equations

$$\begin{aligned}&u_t-a\,\delta u^{a-1}u_x+\alpha b(b-1)u^{b-2}(u_x)^3+4\alpha bu^{b-1}u_xu_{2x}\\&+2\alpha u^bu_{3x}+3\beta c(c-1)u^{c-2}(u_x)^5\\&+24\beta cu^{c-1}(u_x)^3u_{2x}+24\beta u^cu_x(u_{2x})^2\\&+12\beta u^c(u_x)^2u_{3x}-2\gamma d(d-1)(d-2)u^{d-3}(u_x)^3u_{2x}\\&-7\gamma d(d-1)u^{d-2}u_x(u_{2x})^2-6\gamma d(d-1)u^{d-2}(u_x)^2u_{3x}\\&-10\gamma du^{d-1}u_{2x}u_{3x}-6\gamma du^{d-1}u_xu_{4x}-2\gamma u^du_{5x}=0,\end{aligned} \tag{18}$$

where $u(x)=\partial_x\phi(x)$. This equation has the same terms as in the Eq. (2), but the relative weights of the terms are different, leading to more than one conservation law for the $K(m,n,p)$ type equations. For the sake of comparison, the set of parameters m,n,p in Eq. (2) corresponds to $a=m$, $b+1=n$, and $c+3=d+1=p$ in Eq. (18). The equation corresponding to the case of $p=2$ of Eq. (2) can be obtained from Eq. (18) by putting $\beta=0$.

We can now obtain two conservation laws for Eq. (18) as

$$Q_1=u,$$

$$\begin{aligned}X_1=&-\delta u^a+\alpha b u^{b-1}(u_x)^2+2\alpha u^b u_{2x}\\&+3\beta c u^{c-1}(u_x)^4+12\beta u^c(u_x)^2u_{2x}\\&-3\gamma d u^{d-1}(u_{2x})^2-2\gamma d(d-1)u^{d-2}(u_x)^2u_{2x}\\&-4\gamma d u^{d-1}u_x u_{3x}-2\gamma u^d u_{4x}\end{aligned}\tag{19}$$

and

$$Q_2=\frac{u^2}{2},$$

$$\begin{aligned}X_2=&\alpha(b-1)u^b(u_x)^2+2\alpha u^{b+1}u_{2x}+3\beta(c-1)u^c(u_x)^4\\&+12\beta u^{c+1}(u_x)^2u_{2x}-(3d+1)\gamma u^d(u_{2x})^2\\&-2\gamma d(d-2)u^{d-1}(u_x)^2u_{2x}-2\gamma u^{d+1}u_{4x}\\&-2(2d-1)\gamma u^d u_x u_{3x}.\end{aligned}\tag{20}$$

The third conserved quantity Q_3 can also be obtained as the Hamiltonian H, which is obtained from the Lagrangian [Eq. (17)] as

$$\begin{aligned}H=&\int[\pi\dot{\phi}-\mathcal{L}]dx,\qquad \pi=\frac{\partial\mathcal{L}}{\partial\dot{\phi}}=\frac{1}{2}\phi_x\\=&\int\left[\delta\frac{u^{a+1}}{a+1}+\alpha u^b(u_x)^2+\beta u^c(u_x)^4+\gamma u^d(u_{2x})^2\right]dx.\end{aligned}\tag{21}$$

This third conserved quantity follows from the fact that Eq. (18) can be written in the canonical form

$$u_t=\partial_x\frac{\delta H}{\delta u}=\{u,H\},\tag{22}$$

where the Poisson bracket structure is given by

$$\{u(x),u(y)\}=\partial_x\delta(x-y).\tag{23}$$

The system of Eqs. (18) thus has the first three conservation laws, similar to that of the usual KdV-type equations, namely, the

$$\text{``area'' under:}\quad u(x,t)=\int_{-\infty}^{+\infty}u(x,t)dx,\tag{24}$$

$$\text{``mass'':}\quad M=\frac{1}{2}\int_{-\infty}^{+\infty}u^2(x,t)dx,\tag{25}$$

and

$$\begin{aligned}\text{``energy'':}\quad E=&H\\=&\int_{-\infty}^{+\infty}\left[\delta\frac{u^{a+1}}{a+1}+\alpha u^b(u_x)^2+\beta u^c(u_x)^4\right.\\&\left.+\gamma u^d(u_{2x})^2\right]dx.\end{aligned}\tag{26}$$

We could not find any other conservation laws for the system of equations (18). Our inability to find more conservation laws may suggest that the systems of equations (18) may not be integrable.

Equations (18) support a class of one parameter family of compacton solutions of the form

$$u(\xi)=A\cos^{\nu}(B\xi),\tag{27}$$

where

$$\nu=\frac{4}{k-1},\tag{28}$$

$$A^{k-1}=\frac{D}{B^4\nu\{3\beta(k+1)\nu^3-12\beta\nu^2-\gamma[\nu^3(2k^2+k-1)-2\nu^2(k^2+6k-1)+11\nu(k+1)-12]\}},\tag{29}$$

$$B^2=\frac{\alpha[2-(k+1)\nu]}{2\gamma[\nu^3(2k^2+k-1)-\nu^2(k^2+6k-1)+4\nu(k+1)-4]+12\beta\nu^2-6\beta(k+1)\nu^3},\tag{30}$$

$$\gamma=\frac{\delta+B^2\nu^2[\alpha(k+1)-3\beta B^2\nu^2(k+1)]}{B^4\nu^4(2k^2+k-1)},\tag{31}$$

and

$$|B\xi|\leqslant\pi/2.\tag{32}$$

$u(\xi)$ is zero otherwise. From Eq. (30) we see that the width of the compacton is independent of the amplitude and velocity D. The compacton solutions exist for continuous values of the nonlinearity parameter $k=a=b+1=c+3=d+1$ in the range $2\leqslant k\leqslant 5$. Note that the nonlinearity parameter k corresponds to $k=m=n=p$ in Eq. (2) for the compacton solutions. For $k=2,3,5$ we get back the three compacton solutions for the integer values of the parameter m,n,p of Eq. (2) as obtained earlier [Eqs. (12)–(14)]. The constants A_k and B_k can be obtained from Eqs. (28)–(31) by substituting $k=2$, 3, and 5, respectively. The compacton solution corresponding to the case

$$K(2,2,2)\ \text{equation:}\quad u(\xi)=A_2\cos^4(B_2\xi) \tag{33}$$

is a new solution of the fifth order equation which is not there for the third-order equation [Eq. (1)] [1]. We can obtain the ''mass'' and ''energy'' for the family of compacton solutions represented by Eq. (27). As an example we obtain the ''mass'' and ''energy'' of the new compacton solution [Eq. (33)] of the $K(2,2,2)$ equation as

$$m=\frac{35\pi(A_2)^2}{256B_2} \tag{34}$$

and

$$E=\frac{35\pi A_2{}^3}{128B_2}\left[\frac{11}{40}+\frac{131}{35}\alpha B_2{}^2+\frac{48}{5}(22\beta-3\gamma)B_2{}^4\right], \tag{35}$$

where A_2 and B_2 can be obtained from Eqs. (28)–(31) by substituting $k=2$. Similarly one can obtain the ''mass'' and ''energy'' corresponding to compacton solutions for other allowed values of the parameter k. In the next section we show how to obtain a class of exact compacton solutions of the generalized arbitrary odd-order KdV equations [6].

IV. COMPACTON SOLUTIONS OF THE GENERALIZED ODD-ORDER KdV EQUATIONS

We consider the generalized two parameter arbitrary odd-order KdV equation of the form

$$u_t+\beta_1(u^k)_x+\sum_{n=1}^{n}\beta_{n+1}(u^k)_{(2n+1)x}=0, \tag{36}$$

where $\beta_i(i=1,2,3,\ldots)$ are real numbers. Using the transformation $\xi=x-Dt$ the above equation reduces to

$$-Du_\xi+\beta_1(u^k)_\xi+\sum_{n=1}^{n}(u^k)_{(2n+1)\xi}=0 \tag{37}$$

using the ansatz for a class of compacton solutions

$$u(\xi)=A\cos^\delta(B\xi) \tag{38}$$

for $|B\xi|\leqslant\pi/2$ and $u(\xi)=0$ otherwise for the compacton solutions, we obtain

$$(u^k)_{3\xi}=kA^{k-1}B^2[(\delta k-1)(\delta k-2)\cos^{\delta(k-1)-2}(B\xi) -(\delta k)^2\cos^{\delta(k-1)}(B\xi)]u_\xi, \tag{39}$$

$$(u^k)_{5\xi}=kA^{k-1}B^4[(\delta k-1)(\delta k-2)(\delta k-3)(\delta k-4)\times\cos^{\delta(k-1)-4}(B\xi)-(\delta k-2)(\delta k-1)\times(2\delta^2k^2-4\delta k+4)\cos^{\delta(k-1)-2}(B\xi)+(\delta k)^4\cos^{\delta(k-1)}(B\xi)]u_\xi \tag{40}$$

and in general for any odd order $(n\geqslant3)$ derivative of u^k, it can be shown that

$$(u^k)_{(2n+1)\xi}=kA^{k-1}B^{2n}\Bigg[M_n^{\delta(k-1)-2n}\cos^{\delta(k-1)-2n}(B\xi)-(\delta k-2n+2)M_n^{\delta(k-1)-2n+2}\times\cos^{\delta(k-1)-2n+2}(B\xi)+\sum_{m=2}^{n-2}(-1)^{m+n}\times(\delta k-2m)M_n^{\delta(k-1)-2m}\cos^{\delta(k-1)-2n}(B\xi)+(-1)^{n-1}(\delta k-2)M_n^{\delta(k-1)-2}\cos^{\delta(k-1)-2}\times(B\xi)+(-1)^n(\delta k)^{2n}\cos^{\delta(k-1)}(B\xi)\Bigg]u_\xi, \tag{41}$$

where

$$M_n^{\delta(k-1)-2n}=\prod_{i=1}^{2n}(\delta k-i), \tag{42}$$

$$M_n^{\delta(k-1)-2n+2}=(\delta k-2n+2)\prod_{i=1}^{2n-2}(\delta k-i)+(\delta k-2n+3)\times(\delta k-2n+4)M_{n-1}^{\delta(k-1)-2n+4}, \tag{43}$$

$$M_n^{\delta(k-1)-2m}=(\delta k-2m)^2M_{n-1}^{\delta(k-1)-2m}+(\delta k-2m+2)\times(\delta k-2m+1)M_{n-1}^{\delta(k-1)-2m+2} \tag{44}$$

for $m=2,3,\ldots,n-2$,

$$M_n^{\delta(k-1)-2}=(\delta k-2)^2M_{n-1}^{\delta(k-1)-2}+(\delta k-1)(\delta k)^{2n-2}, \tag{45}$$

with $M_0^{\delta(k-1)-2}=0$, $M_1^{\delta(k-1)-2}=(\delta k-1)$, and $M_1^{\delta(k-1)}=k\delta$. Thus we see that all odd-order derivatives of u^k can be expressed as a product of u_ξ and a function of $u(\xi)$. Thus, Eq. (41) when substituted in Eq. (37) gives us an algebric equation that can be solved. So substituting Eqs. (41)–(45) in Eq. (37) and equating the coefficients of various powers of $u(\xi)$ to zero, we get a set of equations that can be solved for the values of A, B, δ in Eq. (38) as well as the relations between the β_i's under which the two parameter equations [Eq. (36)] has the compacton solutions. We give below some examples for different values of the nonlinearity parameter k and odd-order dispersion parameter n. For $n=2$ ($n=1$ and $k=2$ case [Eq. (1) is considered in [1]] and k arbitrary, Eq. (36) reduces to

$$u_t+\beta_1(u^k)_x+\beta_2(u^k)_{3x}+\beta_3(u^k)_{5x}=0. \tag{46}$$

As shown earlier, for $m=n=p=k$ Eq. (2) supports compacton solutions [Eqs. (7)–(11)] and this equation is the same as Eq. (46) above. Substituting Eqs. (38)–(40) in Eq. (46) we get the set of equations for

$$\delta=\frac{4}{k-1} \tag{47}$$

as

$$D-kA^{k-1}B^4\beta_3(\delta k-1)(\delta k-2)(\delta k-3)(\delta k-4)=0, \tag{48}$$

$$\beta_1-(k\delta B)^2\beta_2+(k\delta B)^4\beta_3=0, \tag{49}$$

$$\beta_2-\beta_3[(k\delta-2)^2+(k\delta)^2]=0. \tag{50}$$

Now from Eq. (47) we see that the class of compacton solutions is obtained for nonlinearity parameter k in the range $2\leqslant k\leqslant 5$. Thus we have three compacton solutions for integer values of the nonlinearity parameter $k=2,3,5$ which can be obtained from Eq. (38) and Eqs. (47)–(50) by substituting a corresponding value of the parameter k. It can be easily checked that these solutions agree with the solutions in Eqs. (12)–(14). It should be noted that the class of compacton solutions exists for all continuous (both integer and noninteger) values of the nonlinearity parameter k in the range $2\leqslant k\leqslant 5$. Similarly for any arbitrary odd-order dispersion parameter $n\geqslant 3$, we get from Eq. (37) and Eqs. (41)–(45) for

$$\delta=\frac{2n}{k-1} \tag{51}$$

the set of equations for determining A,B and relation between β_i's as

$$D-kA^{k-1}B^{2n}\beta_{n+1}M_n^{\delta(k-1)-2n}=0,$$

$$\sum_{p=0}^{n}(-1)^p(k\delta B)^{2p}\beta_{p+1}=0,$$

$$\sum_{p=1}^{n}(-1)^{p-1}(B)^{2p-2}\beta_{p+1}M_p^{\delta(k-1)-2}=0,$$

$$\beta_3M_2^{\delta(k-1)-4}+(\delta k-4)\sum_{p=3}^{n}(-1)^p\beta_{p+1}B^{2p-4}M_p^{\delta(k-1)-4}=0,$$

$$\vdots \quad \vdots \quad \vdots$$

$$\vdots \quad \vdots \quad \vdots$$

$$\beta_{n-1}M_{n-2}^{\delta(k-1)-2n+4}-(\delta k-2n+4)(\beta_nB^2M_{n-1}^{\delta(k-1)-2n+4}-\beta_{n+1}B^4M_n^{\delta(k-1)-2n+4})=0,$$

$$\beta_nM_{n-1}^{\delta(k-1)-2n+2}-\beta_{n+1}B^2(\delta k-2n+2)M_n^{\delta(k-1)-2n+2}=0. \tag{52}$$

For a given value of the nonlinearity parameter k and the odd-order dispersion parameter n, one can find δ from Eq. (51) and other values of constants A,B and β_i's from Eq. (52) to obtain the compacton solutions [Eq. (38)] of the given Eq. (36). As an example, for the $n=3$ case, the equation is

$$u_t+\beta_1(u^k)_x+\beta_2(u^k)_{3x}+\beta_3(u^k)_{5x}+\beta_4(u^k)_{7x}=0. \tag{53}$$

From Eq. (51) we see that the compacton solutions [Eq. (38)] now exist for the nonlinearity parameter k within the range $2\leqslant k\leqslant 7$. From Eq. (52) we get the corresponding set of equations as

$$D-kA^{k-1}B^6\beta_4\prod_{i=1}^{6}(\delta k-i)=0, \tag{54}$$

$$\beta_1-(k\delta B)^2\beta_2+(k\delta B)^4\beta_3+(k\delta B)^6\beta_4=0, \tag{55}$$

$$\beta_2-2B^2\beta_3(\delta^2k^2-2\delta k+2)+B^4\beta_4[(2k^2\delta^2-4k\delta+4)\times(k\delta-2)^2+(k\delta)^4]=0, \tag{56}$$

$$\beta_3-B^2\beta_4(3k^2\delta^2-12k\delta+20)=0, \tag{57}$$

which can be solved for A,B and the relation between β_i's to get the compacton solutions.

Thus we find that with an increase in the value of the odd-order dispersion parameter n, the range of the nonlinearity parameter k also increases. From Eq. (51) we find that for a given value of parameter n, the class of compacton solutions is allowed for parameter k in the range $2\leqslant k\leqslant(2n+1)$. Similarly the value of δ [Eq. (38)], for which the compacton solutions are allowed, also increases with the increase in parameter n. For a given value of the parameter n, the value of δ varies within the range $1\leqslant\delta\leqslant 2n$.

V. CONCLUSIONS

We have examined here the effect of a higher-order dispersion term on a particular class of compacton solutions of the $K(m,n)$ type equations as studied by Rosenau and Hyman [1]. We find that the addition of the higher-order dispersion term increases the range of the nonlinearity parameter for which the class of compacton solutions are allowed. We have studied the Hamiltonian structure and conservation laws for these types of higher-order equations and find that these equations may not be integrable. We generalize the problem by studying the two parameter generalized equations with an arbitrary odd-order dispersion term and nonlinearity and obtain a class of exact compacton solutions of this generalized system of equations. We also obtain the range of nonlinear parameters as well as the relations between them for which the compacton solutions are allowed for this generalized system of equations. It would be interesting to examine the stability of these compacton solutions as obtained here to find whether the compacton solutions for all the continuous values of the parameters within their allowed range are stable, or, the parameter range as obtained here is further limited by the stability criteria for the compacton solutions, as it happens in the case of soliton stability for the higher-order KdV type equations [4,5]. This is, however, an independent problem and requires finding analytically stability criteria for the compacton solutions, as is done for the case of soliton stability . The other alternative approach would be to perform numerical simulations of the scattering of compacton solutions [1] to examine the stability and preservation of shape after scattering.

[1] P. Rosenau and J. M. Hyman, Phys. Rev. Lett. **70**, 564 (1993).
[2] N. J. Zabusky and M. D. Kruskal, Phys. Rev. Lett. **15**, 240 (1965).
[3] F. Cooper, H. Shepard, and P. Sodono, Phys. Rev. E **48**, 4027 (1993); A. Khare and F. Cooper, *ibid.* **48**, 4843 (1993).
[4] V. I. Karpman, Phys. Lett. A **210**, 77 (1996), and references therein.
[5] B. Dey, A. Khare, and C. N. Kumar, Phys. Lett. A **223**, 449 (1996).
[6] Z. J. Yang, Int. J. Theor. Phys. **34**, 641 (1995).